

El-Gamal Encryption Scheme

p - large prime
 $g \in [2, p-2]$ - generator
 g^b - Bob's public key

m - Alice

$$r \xleftarrow{\$} [1, p-1]$$

$$(g^r, (g^b)^r \cdot m)$$

Bob - b

$$\frac{c_2}{(c_1)^b} = \frac{g^{br} \cdot m}{g^{rb}} \equiv m$$

- We know discrete log is hard... how to build encryption from it?

... Embed message in exponent? ie. g^m

→ This hides the message but isn't decryptable

- We want something like $m \cdot k$ where k is only known to Alice & Bob

→ This is just a OTP!

Idea: Use DH Key exch. to create a new k for every ciphertext

For each encryption:

- $k = g^{br}$ → Alice can compute since she knows r & g^b
- This is DH key exch. where g^b is static

- $c = (g^r, k \cdot m)$ → Bob can compute k & decrypt since he knows g^r & b

⇒ El-Gamal Encryption can be thought as a OTP where the key is randomly generated on each encryption via DH Key Exch.